



IV Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

Valencia, 4-8 de septiembre de 2017

Cubic fourfolds and non-commutative K3 surfaces

A. Bayer^{*} M. Lahoz[†] E. Macrì[‡] H. Nuer[§] A. Perry[¶] P. Stellari^{||}

The derived category of coherent sheaves on a smooth cubic fourfold has a subcategory, called the Kuznetsov component, that can be thought as the derived category of a non-commutative K3 surface. This category has recently been studied by Kuznetsov and Addington–Thomas, among others. Namely, it is conjecturally [6, Conjecture 1.1] the derived category of a K3 surface if and only if the cubic fourfold is rational, and this conjecture is equivalent (generically) to a folklore conjecture characterizing rationality of a cubic fourfold in terms of its Hodge structure [1, Theorem 1.1].

Moreover, the most interesting part of the geometry of moduli space of sheaves on a cubic fourfold X is induced by the Kuznetsov component $\mathcal{K}u(X)$ —which explains the interest in the question on the existence of stability conditions on $\mathcal{K}u(X)$.

In this talk, I will present joint work with Bayer, Macrì, and Stellari [2] about the construction of Bridgeland stability conditions on the Kuznetsov component. This allows in particular to recover the 8-dimensional hyperkähler variety associated to cubic fourfolds X via the Hilbert scheme of twisted cubics [8] as a moduli space of Bridgeland stable complexes in $\mathcal{K}u(X)$ [7] (for X very general).

If time permits, I will also explain the notion of stability condition over a curve, which we have introduced with Bayer, Macrì, Nuer, and Perry (it will appear in [3]). Thanks to this notion, we can use deformation arguments to study moduli spaces on $\mathcal{K}u(X)$ and extend the fundamental results (due to Mukai, Huybrechts, O’Grady, Yoshioka, Toda [5, 9, 11, 10]) on non-emptiness of moduli spaces of sheaves and stable objects on K3 surfaces to stable objects in $\mathcal{K}u(X)$ (work in progress together with Bayer, Macrì, and Stellari [4]). This allow us to naturally improve and complete the aforementioned result by

Addington–Thomas [1].

References

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^{*}School of Mathematics and Maxwell Institute, University of Edinburgh, Peter Guthrie Tait Road, Edinburgh, EH9 3FD, United Kingdom.

[†]Universitat de Barcelona, Gran Via de les Corts Catalanes, 585, 08007 Barcelona &

Université Paris Diderot – Paris 7, Institut de Mathématiques Jussieu - Paris Rive Gauche, Bâtiment Sophie-Germain, F-75205 Paris.

Email: marti.lahoz@imj-prg.fr

[‡]Department of Mathematics, Northeastern University, 360 Huntington Avenue, Boston, MA 02115, USA.

[§]Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, 851 S. Morgan Street Chicago, IL 60607.

[¶]Department of Mathematics, Columbia University, 2990 Broadway, New York, NY 10027, USA.

^{||}Dipartimento di Matematica “F. Enriques”, Università degli Studi di Milano, Via Cesare Saldini 50, 20133 Milano, Italy.

